Coherent State Construction of Representations of $osp(2|2)$ and Primary Fields of $osp(2|2)$ Conformal Field Theory

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Abstract

Representations of the superalgebra $osp(2|2)$ and current superalgebra $osp(2|2)^{(1)}_k$ in the standard basis are investigated. All finite-dimensional typical and atypical representations of $osp(2|2)$ are constructed by the vector coherent state method. Primary fields of the non-unitary conformal field theory associated with $osp(2|2)^{(1)}_k$ in the standard basis are obtained for arbitrary level k.

1 Introduction

Superalgebras and their corresponding non-unitary conformal field theories (CFTs) have recently attracted much interests in high energy and condensed matter physics communities, partly because of their applications in areas such as topological field theory [1, 2], logarithmic CFTs (see e.g. [3] and references therein) and disordered systems [4, 5, 6, 7, 8, 9]. There the vanishing of superdimensions and Virasoro central charges and the existence of primary fields with negative dimensions are essential [5, 6, 10, 11]. The most interesting superalgebras satisfying these requirements are $osp(N|N)$ and $gl(N|N)$.

It is well-known that unlike a purely bosonic algebra a superalgebra admits different Weyl inequivalent choices of simple root systems, which correspond to inequivalent Dynkin diagrams. In the case of $osp(2|2)$, one has two choices of simple root systems which are unrelated by Weyl transformations: a system of fermionic and bosonic simple roots (i.e. the so-called standard basis), or a purely fermionic system of simple roots (that is the so-called non-standard basis). So it is desirable to obtain results in the two different bases for different physical applications.

Representations of $osp(2|2)$ have been constructed only in the non-standard basis [12, 13] (see also [14, 15, 16, 17] for the study of certain specific representations). Primary fields of the $osp(2|2)$ non-unitary CFT have been investigated in [18]. However, for the standard basis case the expressions found in [18] are valid only for a class of atypical representations corresponding to $q = p$ (see section 4 below).

In this paper we construct explicitly all finite-dimensional typical and atypical representations of $osp(2|2)$ in the standard basis by using the vector coherent state method. We moreover construct all primary fields in the standard basis for the $osp(2|2)$ non-unitary CFT. The operator product expansions (OPEs) of the $osp(2|2)$ currents with the primary fields are presented. Our results are expected to be useful in the supersymmetric method to certain Gaussian disordered systems.

2 Boson-fermion realizations of $osp(2|2)$

Superalgebra $osp(2|2)$ is a $\mathbf{Z}_2$-graded algebra, $osp(2|2) = osp(2|2)^{\rm even} \oplus osp(2|2)^{\rm odd}$, with

$$osp(2|2)^{\rm even} = u(1) \oplus sl(2) = \{H'\} \oplus \{H, E, F\}, \qquad osp(2|2)^{\rm odd} = \{e, f, \bar{e}, \bar{f}\}, \tag{2.1}$$

where $e,\ f,\ \bar{e},\ \bar{f}$ are the generators corresponding fermionic roots, and $E,\ F$ are those to bosonic roots.

In the standard basis, E (F) and e (f) are the generators corresponding to the even and odd simple roots of $osp(2|2)$, respectively, and $\bar{e},\ \bar{f}$ are the odd non-simple generators. They satisfy the following (anti-)commutation relations:

$$\begin{aligned}
&[E,F] = H, \quad [H,E] = 2E, \quad [H,F] = -2F,\\
&\{e,f\} = -\frac{1}{2}(H - H'), \quad [H,e] = -e, \quad [H,f] = f,\\
&[H',e] = -e, \quad [H',f] = f,\\
&[E,e] = \bar{e}, \quad [F,f] = \bar{f},\\
&\{\bar{e},\bar{f}\} = -\frac{1}{2}(H + H'), \quad [H,\bar{e}] = \bar{e}, \quad [H,\bar{f}] = -\bar{f},\\
&[H',\bar{e}] = -\bar{e}, \quad [H',\bar{f}] = \bar{f},\\
&\{e,\bar{f}\} = -F, \quad \{\bar{e},f\} = E, \quad [E,\bar{f}] = f, \quad [F,\bar{e}] = e.
\end{aligned} \tag{2.2}$$

All other (anti-)commutators are zero. The quadratic Casimir is given by

$$C_2 = \frac{1}{2}\left(H(H+2) - H'(H'+2)\right) + 2fe - 2\bar{f}\bar{e} + 2FE. \tag{2.3}$$

Let $|hw>$ be the highest weight state of highest weight (p,q) of $osp(2|2)$ in the standard basis:

$$H|hw>= 2p|hw>, \qquad H'|hw>= 2q|hw>, \qquad E|hw>= e|hw>= \bar{e}|hw>= 0. \tag{2.4}$$

Define the vector coherent states, $e^{Fa+\alpha_1 f+\bar{f}\alpha_2}|hw>$. As is known from [12], the definition of an (grade) adjoint action on the Lie superalgebra generators is not unique. Therefore the order between the fermionic operators in the exponent defining the coherent states is inmaterial in the sense that it will not affect the conclusions of this paper. Here we have chosen a order such that the dual state has the form appeared in (2.5) below.

Then state vectors $|\psi>$ are mapped into functions

$$\begin{aligned}\psi_{p,q} &= <hw|\exp\left(a^\dagger E + \alpha_1^\dagger e + \alpha_2^\dagger \bar{e}\right)|\psi>|0> \\ &= <hw|e^{a^\dagger E} e^{\alpha_1^\dagger e} e^{(\alpha_2^\dagger - \frac{1}{2}\alpha_1^\dagger a^\dagger)\bar{e}}|\psi>|0>. \end{aligned} \tag{2.5}$$

Here $a, a^\dagger$ are bosonic operators with number operator N_a, and α_1 $(\alpha_1^\dagger), \alpha_2$ $(\alpha_2^\dagger)$ are fermionic operators with number operators $N_{\alpha_1}, N_{\alpha_2}$, respectively. These operators satisfy relations:

$$\begin{aligned} &[a, a^\dagger] = 1, \quad [N_a, a^\dagger] = a^\dagger, \quad [N_a, a] = -a, \\ &\{\alpha_1, \alpha_1^\dagger\} = 1, \quad [N_{\alpha_1}, \alpha_1^\dagger] = \alpha_1^\dagger, \quad [N_{\alpha_1}, \alpha_1] = -\alpha_1, \\ &\{\alpha_2, \alpha_2^\dagger\} = 1, \quad [N_{\alpha_2}, \alpha_2^\dagger] = \alpha_2^\dagger, \quad [N_{\alpha_2}, \alpha_2] = -\alpha_2, \end{aligned} \tag{2.6}$$

all other (anti-)commutators are zero. Moreover, $a|0>=\alpha_1|0>=\alpha_2|0>=0$.

Operators A are mapped as follows

$$A|\psi> \longrightarrow \Gamma(A)\psi_{J,q} = <hw|e^{a^\dagger E} e^{\alpha_1^\dagger e} e^{(\alpha_2^\dagger - \frac{1}{2}\alpha_1^\dagger a^\dagger)\bar{e}} A|\psi>|0>. \tag{2.7}$$

Taking $A = H, H', E, \cdots$ in turn and after some algebraic manipulations, we find

$$\begin{aligned} \Gamma(H) &= 2p - 2N_a + N_{\alpha_1} - N_{\alpha_2}, \quad \Gamma(H') = 2q + N_{\alpha_1} + N_{\alpha_2}, \\ \Gamma(E) &= a - \frac{1}{2}\alpha_1^\dagger \alpha_2, \\ \Gamma(F) &= 2pa^\dagger - \alpha_2^\dagger \alpha_1 - a^\dagger\left(N_a - \frac{1}{2}N_{\alpha_1} + \frac{1}{2}N_{\alpha_2}\right) - \frac{1}{4}(a^\dagger)^2 \alpha_1^\dagger \alpha_2, \\ \Gamma(e) &= \alpha_1 + \frac{1}{2}a^\dagger \alpha_2, \quad \Gamma(f) = -(p-q)\alpha_1^\dagger + \alpha_2^\dagger a + \frac{1}{2}\alpha_1^\dagger\left(N_a + N_{\alpha_2}\right), \\ \Gamma(\bar{e}) &= \alpha_2, \quad \Gamma(\bar{f}) = -(p+q)\alpha_2^\dagger - \frac{1}{2}(3p-q)a^\dagger \alpha_1^\dagger + \alpha_2^\dagger\left(N_a - N_{\alpha_1}\right) + \frac{1}{2}a^\dagger \alpha_1^\dagger N_a. \end{aligned} \tag{2.8}$$

This gives a free boson-fermion realization of $osp(2|2)$ in the standard basis. In this realization, the Casimir takes a constant value, i.e. $C_2 = 2[p(p+1) - q(q+1)]$.

3 Construction of representations of $osp(2|2)$

Unlike ordinary bosonic algebras, there are two types of representations for most superalgebras. They are the so-called typical and atypical representations. The typical representations are irreducible and are similar to the usual representations appeared in ordinary bosonic algebras. The atypical representations have no counterpart in the representation theory of bosonic algebras. They can be irreducible or not fully reducible (i.e. reducible or indecomposable). This makes the study of representations of superalgebras difficult in general.

In this section we use the above free boson-fermion realization to construct finite-dimensional representations of $osp(2|2)$ in the standard bases. As we will see, all finite-dimensional typical and atypical representations of $osp(2|2)$ can be constructed in an unified manner.

To begin, we note that representations of $osp(2|2)$ in the standard basis are labelled by (p,q) with p being a positive integer or half-integer and q an arbitrary complex number. There are four independent combinations of creation operators acting on the vacuum vector $|0>$:

$$
\begin{aligned}
&\left(a^\dagger\right)^{p-m}|0>, \quad p-m\in \mathbf{Z}_+,\\
&\alpha_1^\dagger\left(a^\dagger\right)^{p-m-1/2}|0>, \quad p-\frac{1}{2}-m\in \mathbf{Z}_+,\\
&\alpha_2^\dagger\left(a^\dagger\right)^{p-m-3/2}|0>, \quad p-\frac{3}{2}-m\in \mathbf{Z}_+,\\
&\alpha_1^\dagger\alpha_2^\dagger\left(a^\dagger\right)^{p-m-2}|0>, \quad p-2-m\in \mathbf{Z}_+.
\end{aligned}
\tag{3.1}
$$

Thus each $osp(2|2)$ representation decomposes into at most four representations of the even subalgebra $sl(2)\oplus u(1)$. Let us construct representations for $sl(2)\oplus u(1)$ out of the above states. It is easy to check that the first and the last states are already representations of $sl(2)\oplus u(1)$ with highest weight weights (p,q) and $(p,q+1)$, respectively. We denote these two multiplets by $|p,m;q>$ and $|p,m;q+1>$, respectively. We now show that the second and the third states can be combined into two independent multiplets of $sl(2)\oplus u(1)$ with highest weights $(p-\frac{1}{2},q+\frac{1}{2})$ and $(p+\frac{1}{2},q+\frac{1}{2})$, respectively. Let

$$
\chi^m_{p,q}=\frac{1}{2}c^m_{p,q}\alpha_1^\dagger\left(a^\dagger\right)^{p-m-1/2}|0>+\bar{c}^m_{p,q}\alpha_2^\dagger\left(a^\dagger\right)^{p-m-3/2}|0>,
\tag{3.2}
$$

where $c^m_{p,q}$ and $\bar{c}^m_{p,q}$ are functions of p,q,m to be determined. Then,

$$
\begin{aligned}
\Gamma(E)\chi^m_{p,q} &= \frac{1}{2}\left((p-m-\frac{1}{2})c^m_{p,q}-\bar{c}^m_{p,q}\right)\alpha_1^\dagger\left(a^\dagger\right)^{p-m-3/2}|0>\\
&\quad +(p-m-\frac{3}{2})\bar{c}^m_{p,q}\alpha_2^\dagger\left(a^\dagger\right)^{p-m-5/2}|0>.
\end{aligned}
\tag{3.3}
$$

To make the representation finite-dimensional, the r.h.s. of this equation must equal to $(p-m-\frac{x}{2})\chi^{m+1}_{p,q}$ for some integer x. This requires

$$
\begin{aligned}
(p-m-\frac{x}{2})\bar{c}^{m+1}_{p,q} &= (p-m-\frac{3}{2})\bar{c}^m_{p,q},\\
(p-m-\frac{x}{2})c^{m+1}_{p,q} &= (p-m-\frac{1}{2})c^m_{p,q}-\bar{c}^m_{p,q}.
\end{aligned}
\tag{3.4}
$$

In view of the 2nd and 3rd equations of (3.1), the maximum value that m can have is $p-1/2$ or $p-3/2$. This means that x can only be 1 or 3. So we have two cases to consider:

Case 1: $x=1$, for this case (3.4) has solutions

$$
\bar{c}^m_{p,q}=(p-m-\frac{1}{2})X_{p,q}, \qquad c^m_{p,q}-c^{m+1}_{p,q}=X_{p,q}.
\tag{3.5}
$$

Case 2: $x = 3$, for this case, one has from (3.4)

$$\bar{c}^m_{p,q} = Y_{p,q}, \qquad (p-m-\frac{1}{2})c^m_{p,q} - (p-m-\frac{3}{2})c^{m+1}_{p,q} = Y_{p,q}. \tag{3.6}$$

Here $X_{p,q}$ and $Y_{p,q}$ only depend on p and q. On the other hand,

$$\begin{aligned} \Gamma(F)\chi^m_{p,q} &= \frac{1}{2}\left((p+m+1)c^m_{p,q} - \frac{1}{2}\bar{c}^m_{p,q}\right)\alpha_1^\dagger \left(a^\dagger\right)^{p-m+1/2}|0> \\ &\quad \left((p+m+1)\bar{c}^m_{p,q} - \frac{1}{2}c^m_{p,q}\right)\alpha_2^\dagger \left(a^\dagger\right)^{p-m-1/2}|0> . \end{aligned} \tag{3.7}$$

This must equal to $(p+m+\frac{y}{2})\chi^{m-1}_{p,q}$ for some integer y in order for the representation to be finite-dimensional. So we get

$$\begin{aligned} \frac{1}{2}c^m_{p,q} &= (p+m+1)\bar{c}^m_{p,q} - (p+m+\frac{y}{2})\bar{c}^{m-1}_{p,q}, \\ \frac{1}{2}\bar{c}^m_{p,q} &= (p+m+1)c^m_{p,q} - (p+m+\frac{y}{2})c^{m-1}_{p,q}. \end{aligned} \tag{3.8}$$

Combining (3.8) with (3.5) or (3.6), we obtain

$$y = 3, \quad c^m_{p,q} = 3p+m+\frac{5}{2}, \quad \bar{c}^m_{p,q} = -(p-m-\frac{1}{2}) \tag{3.9}$$

for Case 1, and

$$y = 1, \quad c^m_{p,q} = \bar{c}^m_{p,q} = 1 \tag{3.10}$$

for Case 2. Here we have set the overall factors $X_{p,q}$ and $Y_{p,q}$ to be -1 and 1, respectively. Also it is easily seen that

$$\Gamma(H)\chi^m_{p,q} = 2(m+1)\chi^m_{p,q}, \qquad \Gamma(H')\chi^m_{p,q} = 2(q+\frac{1}{2})\chi^m_{p,q}. \tag{3.11}$$

It follows that $\chi^m_{p,q}$ has highest weight $(p+1/2, q+1/2)$ for Case 1 (where $m_{\max} = p-1/2$) and highest weight $(p-1/2, q+1/2)$ for Case 2 (where $m_{\max} = p-3/2$). This justifies the use of notation, $|p+\frac{1}{2}, m; q+\frac{1}{2}>$ and $|p-\frac{1}{2}, m; q+\frac{1}{2}>$, for these two multiplets, respectively.

Summarizing, we have the following four $sl(2)\oplus u(1)$ multiplets which span finite-dimensional representations of $osp(2|2)$:

$$\begin{aligned} |p,m;q> &= \left(a^\dagger\right)^{p-m}|0>, \quad m = p, p-1, \cdots, -p, \quad p \geq 0 \\ |p-\frac{1}{2}, m; q+\frac{1}{2}> &= \left(\alpha_2^\dagger + \frac{1}{2}\alpha_1^\dagger a^\dagger\right)\left(a^\dagger\right)^{p-3/2-m}|0>, \\ &\quad m = p-\frac{3}{2}, p-\frac{5}{2}, \cdots, -(p+\frac{1}{2}), \quad p \geq \frac{1}{2}, \\ |p+\frac{1}{2}, m; q+\frac{1}{2}> &= \left(p+m+\frac{3}{2}\right)\alpha_1^\dagger\left(a^\dagger\right)^{p-1/2-m}|0> \\ &\quad -\left(p-m-\frac{1}{2}\right)\left(\alpha_2^\dagger - \frac{1}{2}\alpha_1^\dagger a^\dagger\right)\left(a^\dagger\right)^{p-m-3/2}|0>, \\ &\quad m = p-\frac{1}{2}, p-\frac{3}{2}, \cdots, -(p+\frac{3}{2}), \quad p \geq 0, \\ |p,m;q+1> &= \alpha_1^\dagger\alpha_2^\dagger\left(a^\dagger\right)^{p-2-m}|0>, \quad m = p-2, p-3, \cdots, -(p+2), \quad p \geq 0. \end{aligned} \tag{3.12}$$

We remark that the trivial 1-dimensional representation (for which $p = 0 = q$) is provided by $|0>$ and is not included in the above expressions.

The actions of the odd generators on these $sl(2) \oplus u(1)$ multiplets are given by

$$\begin{aligned}
\Gamma(e)|p,m;q> &= 0, \\
\Gamma(f)|p,m;q> &= \frac{p-m}{2p+1}(q+p+1)|p-\frac{1}{2}, m-\frac{1}{2}; q+\frac{1}{2}> \\
&\quad +\frac{1}{2p+1}(q-p)|p+\frac{1}{2}, m-\frac{1}{2}; q+\frac{1}{2}>, \\
\Gamma(\bar{e})|p,m;q> &= 0, \\
\Gamma(\bar{f})|p,m;q> &= -\frac{p+m}{2p+1}(q+p+1)|p-\frac{1}{2}, m-\frac{3}{2}; q+\frac{1}{2}>, \\
&\quad +\frac{1}{2p+1}(q-p)|p+\frac{1}{2}, m-\frac{3}{2}; q+\frac{1}{2}>,
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\Gamma(e)|p-\frac{1}{2}, m; q+\frac{1}{2}> &= |p, m+\frac{1}{2}; q>, \\
\Gamma(f)|p-\frac{1}{2}, m; q+\frac{1}{2}> &= (q-p)|p, m-\frac{1}{2}; q+1>, \\
\Gamma(\bar{e})|p-\frac{1}{2}, m; q+\frac{1}{2}> &= |p, m+\frac{3}{2}; q>, \\
\Gamma(\bar{f})|p-\frac{1}{2}, m; q+\frac{1}{2}> &= (q-p)|p, m-\frac{3}{2}; q+1>,
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\Gamma(e)|p+\frac{1}{2}, m; q+\frac{1}{2}> &= (p+m+\frac{3}{2})|p, m+\frac{1}{2}; q>, \\
\Gamma(f)|p+\frac{1}{2}, m; q+\frac{1}{2}> &= -(p-m-\frac{1}{2})(q+p+1)|p, m-\frac{1}{2}; q+1>, \\
\Gamma(\bar{e})|p+\frac{1}{2}, m; q+\frac{1}{2}> &= -(p-m-\frac{1}{2})|p, m+\frac{3}{2}; q>, \\
\Gamma(\bar{f})|p+\frac{1}{2}, m; q+\frac{1}{2}> &= (p+m+\frac{3}{2})(q+p+1)|p, m-\frac{3}{2}; q+1>,
\end{aligned} \tag{3.15}$$

and

$$\begin{aligned}
\Gamma(e)|p,m;q+1> &= \frac{p+m+2}{2p+1}|p-\frac{1}{2}, m+\frac{1}{2}; q+\frac{1}{2}> \\
&\quad -\frac{1}{2p+1}|p+\frac{1}{2}, m+\frac{1}{2}; q+\frac{1}{2}>, \\
\Gamma(f)|p,m;q+1> &= 0, \\
\Gamma(\bar{e})|p,m;q+1> &= -\frac{p-m-2}{2p+1}|p-\frac{1}{2}, m+\frac{3}{2}; q+\frac{1}{2}> \\
&\quad -\frac{1}{2p+1}|p+\frac{1}{2}, m+\frac{3}{2}; q+\frac{1}{2}>, \\
\Gamma(\bar{f})|p,m;q+1> &= 0,
\end{aligned} \tag{3.16}$$

Note that both $|p,m;q>$ and $|p,m;q+1>$ have dimension $2p+1$, $|p-\frac{1}{2},m;q+\frac{1}{2}>$ has dimension $2p$ and the dimension of $|p+\frac{1}{2},m;q+\frac{1}{2}>$ is $2p+2$. So for $q\neq p,-p-1$, they constitute irreducible typical representation of dimension $8p+4$ of $osp(2|2)$.

When $q=p,\ \ -p-1$, the representations become atypical. We have different types of atypical representations. The Casimir for such representations vanishes, and yet they are not the trivial one-dimensional representation. As can be seen from the actions of odd generators to the $sl(2)\oplus u(1)$ multiplets, for $q=p$, if one starts with $|p,m;q>$ then $|p+\frac{1}{2},m;q+\frac{1}{2}>$ and $|p,m;q+1>$ disappear and only $|p,m;q>$ and $|p-\frac{1}{2},m;q+\frac{1}{2}>$ survive. They form irreducible atypical representation of $osp(2|2)$ of dimension $4p+1$ ($p\geq 1/2$). Similarly, for $q=-p-1$, $|p-\frac{1}{2},m;q+\frac{1}{2}>$ and $|p,m;q+1>$ do not appear and only $|p,m;q>$ and $|p+\frac{1}{2},m;q+\frac{1}{2}>$ remain. They constitute irreducible atypical representation of dimension $4p+3$. Other types of atypical representations are not irreducible. One type of such representations are obtained by starting with $|p,m;q+1>$. As can be seen from the actions of odd generators, these representations contain all multiplets and have dimension $8p+4$. They are not fully reducible and are lowest weight indecomposable Kac modules.

4 Primary fields of $osp(2|2)^{(1)}_k$ in the standard basis

Primary fields are fundamental objects in conformal field theories. A primary field Ψ has the following OPE with the energy-momentum tensor $T(z)$:

$$T(z)\Psi(w)=\frac{\Delta_\Psi}{(z-w)^2}\Psi(w)+\frac{\partial_w\Psi(w)}{z-w}+\ldots, \tag{4.1}$$

where the Δ_Ψ is the conformal dimension of Ψ. Moreover the OPEs of Ψ with the affine currents do not contain poles higher than first order. A special kind of the primary fields is highest weight state.

The current superalgebra $osp(2|2)^{(1)}_k$ in the standard basis can be written as

$$J_A(z)J_B(w)=k\frac{\mathrm{str}(AB)}{(z-w)^2}+f^C_{AB}\frac{J_C(w)}{z-w}, \tag{4.2}$$

where $J_A(z)$ stands for the current of $osp(2|2)^{(1)}_k$ corresponding to the $osp(2|2)$ generator A and f^C_{AB} are structure constants related to $osp(2|2)$ generators A,B and C, which can be read off from their (anti-)commutation relations. In the following, we shall simply use $A(z)$ to denote $J_A(z)$.

Introduce one bosonic β-γ pair, two fermionic b-c type systems and two free scalar fields. These free fields have the following OPEs:

$$\beta(z)\gamma(w)=-\gamma(z)\beta(w)=\frac{1}{z-w},\quad \psi(z)\psi^\dagger(w)=\psi^\dagger(z)\psi(w)=\frac{1}{z-w},$$

$$\bar{\psi}(z)\bar{\psi}^{\dagger}(w)=\bar{\psi}^{\dagger}(z)\bar{\psi}(w)=\frac{1}{z-w},\qquad \phi(z)\phi(w)=-\ln(z-w)=\phi'(z)\phi'(w). \tag{4.3}$$

Then the free field realization of the currents is given by [19, 20, 18]

$$\begin{aligned}
E(z)&=\beta(z)-\frac{1}{2}\psi(z)\bar{\psi}^{\dagger}(z),\\
F(z)&=-i2\alpha_+\partial\phi(z)\gamma(z)-\beta(z)\gamma^2(z)-\bar{\psi}(z)\psi^{\dagger}(z)+\frac{1}{2}\gamma(z)(\psi(z)\psi^{\dagger}(z)-\bar{\psi}(z)\bar{\psi}^{\dagger}(z))\\
&\quad-\frac{1}{4}\gamma^2(z)\psi(z)\bar{\psi}^{\dagger}(z)+(k-\frac{1}{2})\partial\gamma(z),\\
H(z)&=i2\alpha_+\partial\phi(z)-2\beta(z)\gamma(z)+\psi(z)\psi^{\dagger}(z)-\bar{\psi}(z)\bar{\psi}^{\dagger}(z),\\
H'(z)&=2\alpha_+\partial\phi'(z)+\psi(z)\psi^{\dagger}(z)+\bar{\psi}(z)\bar{\psi}^{\dagger}(z),\\
e(z)&=\psi^{\dagger}(z)+\frac{1}{2}\gamma(z)\bar{\psi}^{\dagger}(z),\\
f(z)&=-\alpha_+(i\partial\phi(z)-\partial\phi'(z))\psi(z)+\beta(z)\bar{\psi}(z)+\frac{1}{2}\beta(z)\gamma(z)\psi(z)\\
&\quad+\frac{1}{2}\bar{\psi}(z)\bar{\psi}^{\dagger}(z)\psi(z)+(k+\frac{1}{2})\partial\psi(z),\\
\bar{e}(z)&=\bar{\psi}^{\dagger}(z),\\
\bar{f}(z)&=-\alpha_+(i\partial\phi(z)+\partial\phi'(z))\bar{\psi}(z)-\frac{1}{2}\alpha_+(3i\partial\phi(z)-\partial\phi'(z))\gamma(z)\psi(z)\\
&\quad+\beta(z)\gamma(z)\bar{\psi}(z)-\psi(z)\psi^{\dagger}(z)\bar{\psi}(z)+\frac{1}{2}\beta(z)\gamma^2(z)\psi(z)\\
&\quad-k\partial\bar{\psi}(z)-\frac{1}{2}(k-1)\psi(z)\partial\gamma(z)+\frac{1}{2}(k+1)\gamma(z)\partial\psi(z),
\end{aligned} \tag{4.4}$$

where $\alpha_+=\sqrt{\frac{k+1}{2}}$, and normal ordering is implied in the expressions. In terms of the free fields, the energy-momentum tensor in the standard basis reads

$$\begin{aligned}
T(z)\;&=\;\beta(z)\gamma(z)-\psi^{\dagger}(z)\partial\psi(z)-\bar{\psi}^{\dagger}(z)\partial\bar{\psi}(z)\\
&\quad+\frac{1}{2}\left([i\partial\phi(z)]^2+[\partial\phi'(z)]^2\right)-\frac{1}{\alpha_+}\left(i\partial^2\phi(z)-\partial^2\phi(z)\right).
\end{aligned} \tag{4.5}$$

Now we construct primary fields of the $osp(2|2)$ CFT in the standard basis. It is easy to see that the field

$$V_{p,q}(z)=\exp\{\frac{1}{\alpha_+}(pi\phi(z)-q\phi'(z))\}, \tag{4.6}$$

where p is a positive integer or half-integer and q an arbitrary complex number which specify the representation, is a highest weight state of the $osp(2|2)$ current superalgebra. The conformal dimension of this field is

$$\Delta_{p,q}=\frac{p(p+1)-q(q+1)}{k+1}. \tag{4.7}$$

If $q\neq p,-p-1$, then $\Delta_{p,q}\neq 0$ and the corresponding representations are typical. When $q=p,-p-1$, we have $\Delta_{p,q}=0$ and atypical representations arise. From (3.12), one can

show that the full set of primary fields labelled by p, q are given by

$$
\begin{aligned}
S^m_{p,q}(z) &= \gamma(z)^{p-m} V_{p,q}(z), \quad m = p,\ p-1, \cdots, -(p-1), -p, \quad p \geq 0, \\
s^n_{p,q}(z) &= \gamma(z)^{(p-3/2)-n} \left(\bar{\psi}(z) + \frac{1}{2}\gamma(z)\psi(z) \right) V_{p,q}(z), \\
& \quad n = (p-3/2), \cdots, -(p+1/2), \quad p \geq 1/2, \\
\tilde{s}^l_{p,q}(z) &= \gamma(z)^{(p-3/2)-l} \left((p+l+\frac{3}{2})\gamma(z)\psi(z) - (p-l-\frac{1}{2})[\bar{\psi}(z) - \frac{1}{2}\gamma(z)\psi(z)] \right) V_{p,q}(z), \\
& \quad l = (p-1/2), \cdots, -(p+3/2), \quad p \geq 0, \\
\mathcal{S}^s_{p,q}(z) &= \gamma(z)^{(p-2)-s} \psi(z)\bar{\psi}(z) V_{p,q}(z), \quad s = (p-2), \cdots, -(p+2), \quad p \geq 0. \qquad (4.8)
\end{aligned}
$$

The dimension of both $S^m_{p,q}(z)$ and $\mathcal{S}^s_{p,q}(z)$ is $2p+1$, $\tilde{s}^l_{p,q}(z)$ has dimension $2p$ and $\tilde{s}^l_{p,q}(z)$ is $(2p+2)$-dimensional. So when $q \neq p, -p-1$ the primary fields form an irreducible typical representation of $osp(2|2)$ of dimension $8p+4$. For irreducible atypical representation corresponding to $q = p$, $S^m_{p,q}(z)$ and $s^n_{p,q}(z)$ are the only non-vanishing fields and the dimension of the representation is $4p+1$ ($p \geq 1/2$). For irreducible atypical representation with $q = -p-1$, only $S^m_{p,q}(z)$ and $\tilde{s}^l_{p,q}(z)$ survive and the representation is $(4p+3)$-dimensional.

By means of the free field representations, we may compute the OPEs of $osp(2|2)$ currents with the primary fields. The results are

$$
\begin{aligned}
& E(z)S^m_{p,q}(w) = \frac{p-m}{z-w} S^{m+1}_{p,q}(w), \\
& F(z)S^m_{p,q}(w) = \frac{p+m}{z-w} S^{m-1}_{p,q}(w), \\
& H(z)S^m_{p,q}(w) = \frac{2m}{z-w} S^m_{p,q}(w), \\
& H'(z)S^m_{p,q}(w) = \frac{2q}{z-w} S^m_{p,q}(w), \\
& e(z)S^m_{p,q}(w) = 0, \\
& \bar{e}(z)S^m_{p,q}(w) = 0, \\
& f(z)S^m_{p,q}(w) = \frac{1}{z-w} \left(\frac{p-m}{2p+1}(q+p+1) s^{m-1/2}_{p,q}(w) + \frac{1}{2p+1}(q-p)\tilde{s}^{m-1/2}_{p,q}(w) \right), \\
& \bar{f}(z)S^m_{p,q}(w) = \frac{1}{z-w} \left(-\frac{p+m}{2p+1}(q+p+1) s^{m-3/2}_{p,q}(w) + \frac{1}{2p+1}(q-p)\tilde{s}^{m-3/2}_{p,q}(w) \right),
\end{aligned}
\qquad (4.9)
$$

$$
\begin{aligned}
E(z)s^n_{p,q}(w) &= \frac{(p-3/2)-n}{z-w} s^{n+1}_{p,q}(w), \\
F(z)s^n_{p,q}(w) &= \frac{(p+1/2)+n}{z-w} s^{n-1}_{p,q}(w), \\
H(z)s^n_{p,q}(w) &= \frac{2(n+1)}{z-w} s^n_{p,q}(w),
\end{aligned}
$$

$$
\begin{aligned}
H'(z)s^n_{p,q}(w) &= \frac{2(q+1/2)}{z-w}s^n_{p,q}(w),\\
e(z)s^n_{p,q}(w) &= \frac{1}{z-w}S^{n+1/2}_{p,q}(w),\\
\bar{e}(z)s^n_{p,q}(w) &= \frac{1}{z-w}S^{n+3/2}_{p,q}(w),\\
f(z)s^n_{p,q}(w) &= \frac{q-p}{z-w}\mathcal{S}^{n-1/2}_{p,q}(w),\\
\bar{f}(z)s^n_{p,q}(w) &= \frac{q-p}{z-w}\mathcal{S}^{n-3/2}_{p,q}(w),
\end{aligned} \tag{4.10}
$$

$$
\begin{aligned}
E(z)\tilde{s}^l_{p,q}(w) &= \frac{p-l-1/2}{z-w}\tilde{s}^{l+1}_{p,q}(w),\\
F(z)\tilde{s}^l_{p,q}(w) &= \frac{(p+3/2)+l}{z-w}\tilde{s}^{l-1}_{p,q}(w),\\
H(z)\tilde{s}^l_{p,q}(w) &= \frac{2(l+1)}{z-w}\tilde{s}^l_{p,q}(w),\\
H'(z)\tilde{s}^l_{p,q}(w) &= \frac{2(q+1/2)}{z-w}\tilde{s}^l_{p,q}(w),\\
e(z)\tilde{s}^l_{p,q}(w) &= \frac{p+l+3/2}{z-w}S^{l+1/2}_{p,q}(w),\\
\bar{e}(z)\tilde{s}^l_{p,q}(w) &= -\frac{p-l-1/2}{z-w}S^{l+3/2}_{p,q}(w),\\
f(z)\tilde{s}^l_{p,q}(w) &= -\frac{p-l-1/2}{z-w}(q+p+1)\mathcal{S}^{l-1/2}_{p,q}(w),\\
\bar{f}(z)\tilde{s}^l_{p,q}(w) &= \frac{p+l+3/2}{z-w}(q+p+1)\mathcal{S}^{l-3/2}_{p,q}(w),
\end{aligned} \tag{4.11}
$$

$$
\begin{aligned}
E(z)\mathcal{S}^s_{p,q}(w) &= \frac{(p-2)-s}{z-w}\mathcal{S}^{s+1}_{p,q}(w),\\
F(z)\mathcal{S}^s_{p,q}(w) &= \frac{(p+2)+s}{z-w}\mathcal{S}^{s-1}_{p,q}(w),\\
H(z)\mathcal{S}^s_{p,q}(w) &= \frac{2(s+2)}{z-w}\mathcal{S}^s_{p,q}(w),\\
H'(z)\mathcal{S}^s_{p,q}(w) &= \frac{2(q+1)}{z-w}\mathcal{S}^s_{p,q}(w),\\
e(z)\mathcal{S}^s_{p,q}(w) &= \frac{1}{z-w}\left(\frac{p+s+2}{2p+1}s^{s+1/2}_{p,q}(w) - \frac{1}{2p+1}\tilde{s}^{s+1/2}_{p,q}(w)\right),\\
\bar{e}(z)\mathcal{S}^s_{p,q}(w) &= -\frac{1}{z-w}\left(\frac{p-s-2}{2p+1}s^{s+3/2}_{p,q}(w) + \frac{1}{2p+1}\tilde{s}^{s+3/2}_{p,q}(w)\right),\\
f(z)\mathcal{S}^s_{p,q}(w) &= 0,\\
\bar{f}(z)\mathcal{S}^s_{p,q}(w) &= 0.
\end{aligned} \tag{4.12}
$$

5 Conclusions

We have studied the representations of $osp(2|2)$ in the standard basis by means of the coherent state method. All finite-dimensional typical and atypical representations in this basis have been constructed explicitly. In doing so, we have obtained a free boson-fermion realization of the $osp(2|2)$ algebra. We have also investigated the CFT associated with the current superalgebra $osp(2|2)^{(1)}_k$ in the standard basis. We construct all primary fields corresponding to finite-dimensional representations in the standard basis. The CFT is non-unitary as there exists an infinite family of negative dimensional primary operators in the theory. The procedure presented here for the explicit construction of representations and primary fields may be generalized to other superalgebras.

The coherent state method is also useful in dealing with infinite-dimensional representations of current superalgebras. For example, the boson-fermion realization (2.8) could be used to obtain the free field realization (4.4) of the $osp(2|2)$ currents. This free field realization gives rise to (reducible) infinite-dimensional representations of $osp(2|2)^{(1)}_k$ for $k \neq 0$. Irreducible representations may then be obtained by means of the BRST cohomological analysis, which is out of the scope of the present paper.

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